G045

How to Cope with Smoothing Effect in Ray Based PSDM?

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SUMMARY

Almost all implementations of ray based Kirchhoff prestack depth migration require a smoothed version of the velocity function. However, most velocity model building tools fit prestack traveltimes using blocky parameterizations with finite velocity jumps. This leads to an inconsistent scheme, in the sense that the smoothed migration velocity model is kinematically sub-optimal. Here, using a simple and efficient perturbation method, we show and demonstrate how we can correct for the smoothing effect in Kirchhoff PSDM.

Introduction

Asymptotic ray theory is a powerful and fast method for modelling and imaging applications. The underlying assumptions are valid as long as we can represent seismic data as a combination of seismic events. However, the only way to perform ray based applications on a complex model is to smooth it. In a rough model, the ray paths and behaviours become chaotic (Tappert et al, 1996) and the number of multi-valued arrivals increases excessively. Even worse, some of these arrivals are physically implausible. Kirchhoff PSDM requires a reasonably smoothed velocity model to get stable paraxial quantities (traveltime, amplitude,…), to compute Green's function quickly, and to reduce the error in the interpolation of traveltime tables.

On the other hand, the velocity model is almost always provided by some velocity model builder using a blocky parameterization with finite velocity jumps. This leads us to an inconsistent scheme and some inaccuracies in traveltime tables. Here, we propose to overcome this effect by using a first order perturbation correction of the traveltime tables. The underlying idea of this approach is to use the smoothed version of the velocity model to perform ray tracing while the traveltimes are estimated on the basis of the original unsmoothed velocity model using the frozen ray path. In the following section, we present a short description of the theory and then we demonstrate the proposed method on synthetic and real data.

Theory and Methodology

A Hamiltonian formulation (Goldstein, 1980) simplifies considerably the mathematical analysis of seismic ray theory and the understanding of the physical problem (Burridge, 1976; Farra and Madariaga, 1987). Although not strictly necessary for the following, its relative simplicity and its elegance justify its use. We define the Hamiltonian of ray tracing problem as

$$H(\tau,\vec{x},\vec{p}) = \tfrac{1}{2}\left[{}^{t}\vec{p}.\vec{p} - u^2(\vec{x})\right], \qquad (1)$$

where u denotes the reciprocal of wave propagation velocity, the slowness vector $\vec{p}$ is orthogonal to the wavefront and represents the spatial gradient of traveltime $\vec{p} = \nabla T$, and τ denotes a sampling parameter along the rays such as $dT = \vec{p}\cdot d\vec{x} = u^2(\vec{x})d\tau$. The evolution of the family of ray trajectories is defined by the canonical vector $y(\tau) = (\mathrm{x}(\tau), p(\tau))$, which satisfy the *Hamilton-Jacobi* canonical equations:

$$\begin{aligned} \dot{\vec{x}} &= \nabla_p H = \vec{p} \quad, \\ \dot{\vec{p}} &= -\nabla_x H = \tfrac{1}{2}\nabla_x u^2(\vec{x}) \quad. \end{aligned} \qquad (2)$$

This system of ordinary differential equations is known as the ray tracing system and is much easier to solve numerically than equation (1). The traveltime T can be estimated by integrating the Lagrangian along the ray trajectory such that

$$T = \int_{Ray}\left[{}^{t}\vec{p}\cdot\dot{\vec{x}} - H(\tau,\vec{x},\vec{p})\right]d\tau \cdot \qquad (3)$$

Consider a reference smoothed model characterized by the slowness field $u_0(\vec{x})$ and associated Hamiltonian $H_0 = \frac{1}{2}\left[{}^{t}\vec{p}_0.\vec{p}_0 - u_0^2(\vec{x})\right]$. Integrating the traveltime along the ray path R_0 associated with this reference model, we obtain

$$T_0 = \int_{R_0}\left[{}^{t}\vec{p}_0\cdot\vec{p}_0\right]d\tau = \int_{R_0} u_0^2(\vec{x})d\tau. \qquad (4)$$

Now consider the unsmoothed original velocity model u, which differs slightly from the reference smoothed model u_0, such that $u = u_0 + \Delta u$. This perturbation in slowness (the effect of smoothing in our case) produces a corresponding perturbation of the Hamiltonian $H = H_0 + \Delta H$

where $\Delta H = \frac{\partial H}{\partial u}\Delta u$. The travel time computed on the same frozen ray path R_0 is therefore:

$$T = \int_{R_0}\left[{}^{t}\vec{p}_0 \cdot \left.\frac{\partial H}{\partial \vec{p}}\right|_{u0} - H(\vec{x},\vec{p})\right]d\tau = \int_{R_0}\left[{}^{t}\vec{p}_0 \cdot \left.\frac{\partial (H_0 + \Delta H)}{\partial \vec{p}}\right|_{u0} - \Delta H(\vec{x},\vec{p})\right]d\tau \cdot \qquad (5)$$

In case of first-order approximation, we can write $\frac{\partial (H_0 + \Delta H)}{\partial \vec{p}} \approx \frac{\partial H_0}{\partial \vec{p}}$ and so equation (5) simplifies to

$$T = \int_{R_0}\left[{}^{t}\vec{p}_0 \cdot \vec{p}_0 - \Delta H(\vec{x},\vec{p})\right]d\tau = \int_{R_0}{}^{t}\vec{p}_0 \cdot \vec{p}_0 d\tau - \int_{R_0}\Delta H d\tau \quad . \qquad (6)$$

Analyzing the second part of the second term in the upper equation and remembering that $\Delta H = \partial H/\partial u\,\Delta u$, we obtain

$$\int_{R_0}\Delta H d\tau = -\int_{R_0}u_0(\vec{x})\Delta u d\tau = \int_{R_0}\left[u_0(\vec{x})u(\vec{x}) - u_0^2(\vec{x})\right]d\tau \qquad (7)$$

Finally combining the previous equation and equation (3) the perturbed traveltime associated to the perturbed model (unsmoothed one) is:

$$T = \int_{R_0}u_0(\vec{x})u(\vec{x})d\tau \quad .$$

Application

The first preliminary test was to check/understand the effects of the velocity smoothing for a simple synthetic case, a flat reflector between two homogeneous velocity layers. We generated three 1D blocky models with increasing velocity contrast across the flat interface positioned at 1000 m in depth. Keeping the velocity of the first layer constant to 2000 m/s, we created the models that are characterized by 500 m/s, 1000 m/s and 2000 m/s velocity contrasts. *Kirchhoff* PSDM was applied with the same smoothing parameters (100 m for the radius of smoothing) and imaging. Figure .1. shows on the left CIG result from standard PSDM. The smoothing effect is evident with an increasing velocity contrast and it manifests itself in this example through a residual moveout and a depth shift error. On the right hand side of figure -1-, we show CIG result using traveltime perturbation method. In this case, the reflector remains centered at the correct position and stays almost flat at least up to a critical angle (shown with the star position). We denote also an asymmetric stretch compared to the ideal situation without any contrast (Middle of Figure -1-). This effect is related to the fact that the wavelet, centered across the interface, is influenced by different velocities: lower in the upper part (so less stretched) and higher in the lower part (and so more stretched).

The second synthetic example is the well known SEG-EAGE salt dome. The results obtained with the two different approaches (no perturbations and perturbation of traveltimes) are shown in figure -2-. The main differences are in the top, bottom and some area around the salt body. Both the contacts between the salt body and the sediments are better imaged and positioned with the perturbation approach. Several authors showed similar results (on the same dataset) when they compared *Kirchhoff* PSDM results *vs*. wave equation PSDM and claim that this effect is due to the fact that their *Kirchhoff* imaging algorithm cannot handle multi-valued traveltime arrivals. Our result shows that it solely the effect of velocity model smoothing. Similar conclusion can be made, if we analyze and compare the flatness index (Figure .3.) for the two schemes. The flatness index map shows also that we will be more consistent with interpretation during the iterative velocity model building and that we can speed up the rate of convergence of this iterative strategy.

Real dataset example: The chosen area is a complex structural sub-salt play. Several Salt domes and thin Salt bodies interfere with deformed sediment layers. Traveltime perturbation was tested vs. the standard scheme. The results are presented in figure -4-, where as it was expected, we can see the improvement of continuity and focusing for the top salt. Even more, we can notice

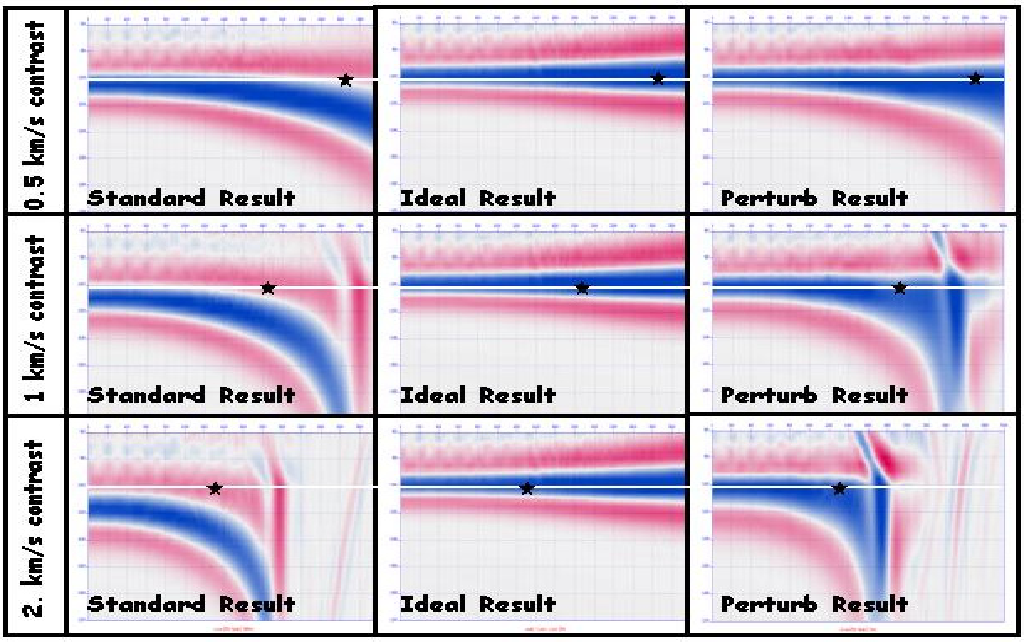

Figure 1: *Comparison of Migrated CIG using standard PSDM (left) and perturbation correction (right) for different velocity contrast. In the middle, Ideal migrated CIG using a homogeneous velocity model is reported.*

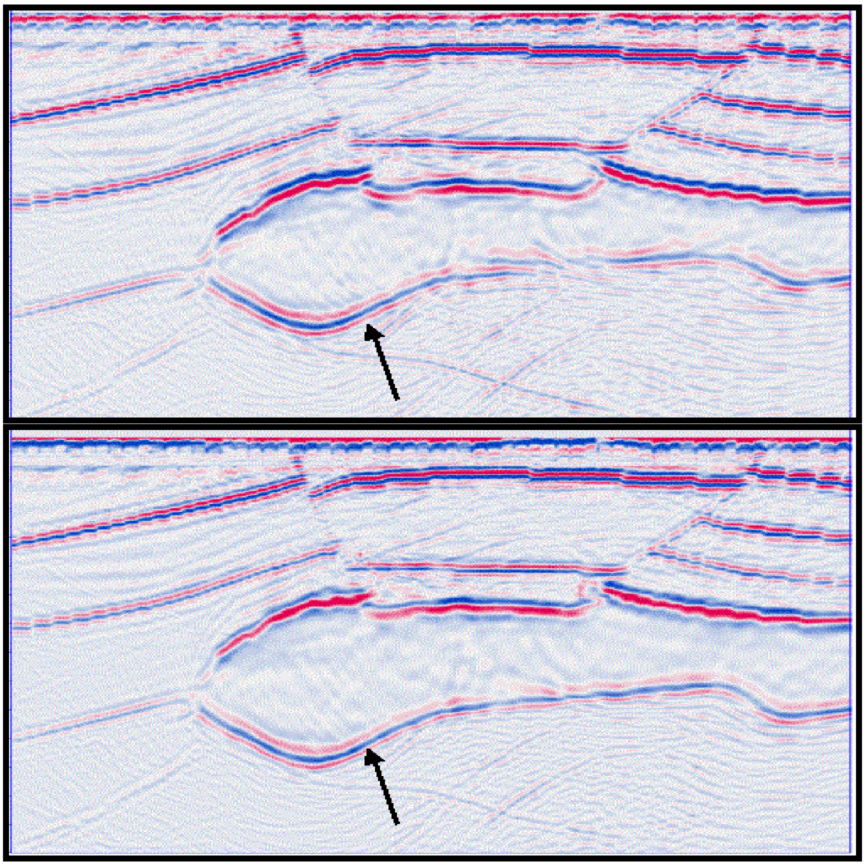

Figure 2: *Top :Depth image using the standard (without perturbation correction) PSDM. Bottom : Depth image using traveltime perturbation scheme.*

the improvement of the character and coherency of faults and structures above the salt body.

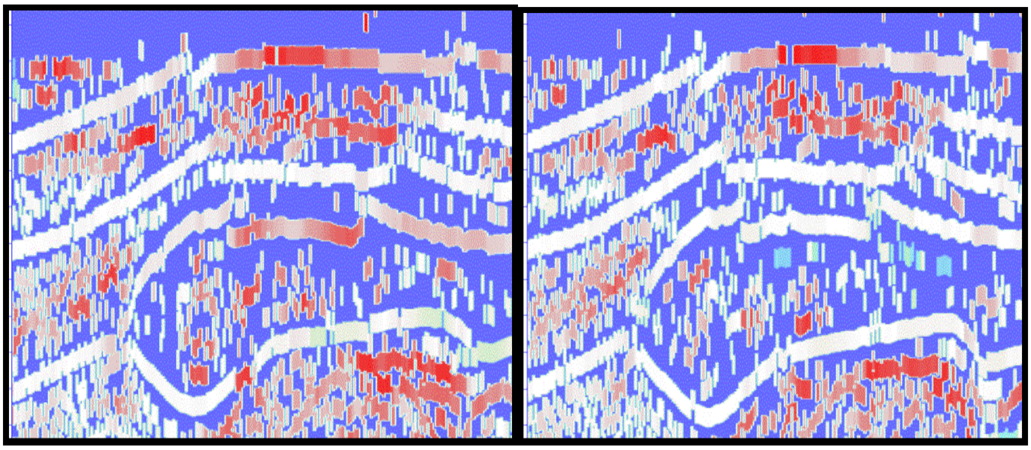

Figure 3: *Flatness indicator of CIG for the standard unperturbed scheme (left) perturbed scheme (right) White color correspond to flat events.*

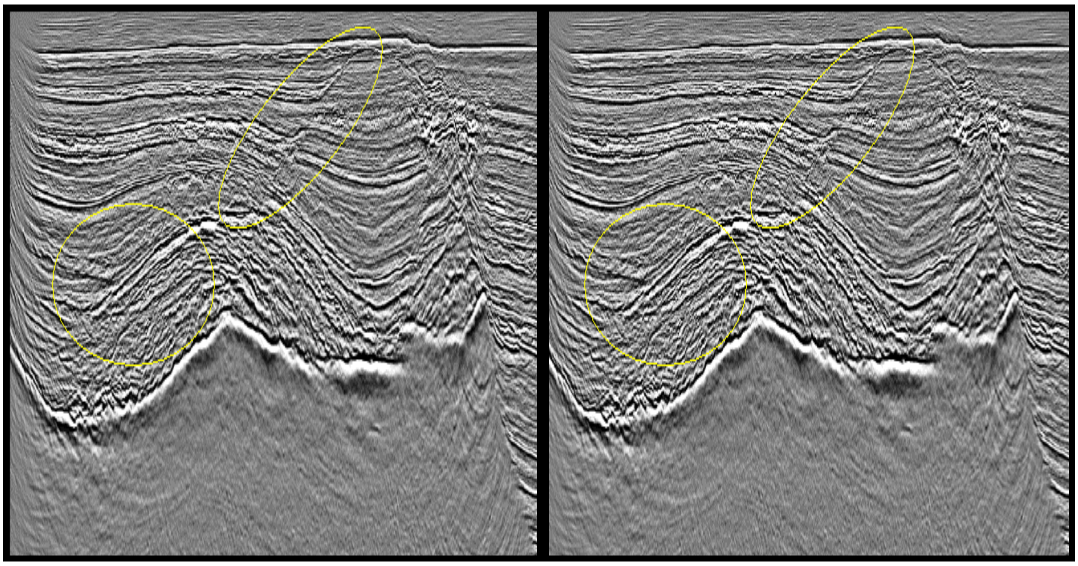

Figure 4: *Comparison of post migration stack using standard scheme (top) and the new traveltime perturbation scheme (bottom).*

Conclusions

We demonstrate in this study how we can cope with smoothing effects in Kirchhoff depth imaging project. The proposed method comes almost without CPU overhead and can be extended to anisotropic media.

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Acknowledgements

Authors want to thank TOTAL for supporting this research. OPERA is a private organization funded by TOTAL and supported by the Pau University whose main objective is to carry out applied research