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3D prediction of surface-related and interbed multiples

Moshe Reshef[1], Shahar Arad[2], and Evgeny Landa[3]

ABSTRACT

Multiple attenuation during data processing does not guarantee a multiple-free final section. Multiple identification plays an important role in seismic interpretation. A target-oriented method for predicting 3D multiples on stacked or migrated cubes in the time domain is presented. The method does not require detailed knowledge of the subsurface geological model or access to prestack data and is valid for both surface-related and interbed multiples.

The computational procedure is based on kinematic properties of the data and uses Fermat's principle to define the multiples. Since no prestack data are required, the method can calculate 3D multiples even when only multi-2D survey data are available. The accuracy and possible use of the method are demonstrated on synthetic and real data examples.

INTRODUCTION

Although the concepts of multiple elimination are the same for both 2D and 3D wavefields, the recent practice of 3D data acquisition does not allow one to apply 3D algorithms directly (van Dedem and Verschuur, 2002) because of the sparseness and irregularity of the acquired data. Even when acquisition parameters are sufficient, the cost of applying a complete 3D multiple suppression procedure may prohibit its use. The current practical implementations of multiple prediction and adaptive subtraction are usually based on 2D approximations (Berryhill and Kim, 1986; Wiggins, 1988; Verschuur et al., 1992; Weglein et al., 1997) and therefore do not guarantee a multiple-free seismic section. In addition, most of the available methods are designed to handle surface-related multiples and ignore interbed multiples. Internal multiples are generally more difficult to remove. The feedback method, which models internal multiples in terms of the actual medium and reflection interfaces that are the sources of those events (Berkhout and Verschuur, 1997), proceeds from one reflector down to the next and removes all internal multiples that have their shallowest reflection at that reflector.

The inverse scattering method for attenuating internal multiples is derived from the multiple prediction and subtraction subseries that reside within the multidimensional direct-inversion methodology (Coates and Weglein, 1996). The cost per reflector of the feedback method is roughly twice the cost of the free-surface algorithm. The cost of the inverse-scattering series approach to internal multiples attenuation is considerably greater.

Development of methods that can recognize 3D multiples on stacked or migrated sections and that do not require direct use of prestack data is of great importance to seismic interpretation. This poststack approach to multiple prediction is considered by Tsai (1985), Kelamis and Verschuur (2000), and Levin (2002). They represent the earth as a 1D stratified medium, and the corresponding stacked section as a plane-wave response. Under these circumstances the multiple-prediction process reduces to an autoconvolution of each stacked trace. This single-trace poststack application is limited because lateral subsurface variations cannot be incorporated in such a crude prediction method.

Reshef et al. (2003) present a 2D method for multiple prediction that can be regarded as a poststack application (i.e., predicting zero-offset arrival times of the multiples, or $T_0^{(m)}$, where T is traveltime). This method, which can predict both surface-related and interbed multiples, is an intermediate solution between a simple 1D single-trace autoconvolution and a full prestack multiple prediction. In other words, the method does not require direct use of prestack data but accounts for lateral velocity variations in the subsurface. Despite the fact that only poststack multiples need to be predicted, the application of this method requires prestack traveltimes of primary reflections from multiple generating interfaces. The necessary prestack traveltimes can be calculated using zero-offset times

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[1]Tel-Aviv University and Landmark Graphics, Ramat Aviv, 69978 Tel-Aviv, Israel. E-mail: moshe@luna.tau.ac.il.
[2]Tel-Aviv University, Department of Geophysics, Ramat Aviv, 69978 Tel-Aviv, Israel.
[3]Organisme Petrolier de Recherche Appliquee (OPERA), Batiment IFR, Rue Jules Ferry, 64000 Pau, France. E-mail: evgeny.landa@univ-pau.fr.

and stacking velocities related to primary events — the multiple generators. In this way the undesirable access of prestack data can be avoided. The method predicts specific multiple events and is highly suitable for interpretive work.

Our study extends the Reshef et al. (2003) method to the 3D case. We show how the principles presented for the 2D case predict 3D multiples without accessing prestack data. Some practical implementation issues are described — in particular, the use of Fermat's principle in the prediction process. The idea of using Fermat's principle to predict surface multiples is presented by Dragoset (2001). We generalize this idea and show how it can be used for interbed multiple prediction. The accuracy and suggested use of the method are demonstrated with synthetic and real data examples.

PREDICTION METHOD

Except for the fact that an additional dimension is added, the procedure we use to predict 3D multiples resembles the 2D case. A detailed description can be found in Reshef et al. (2003).

As with the 2D case, we base our method on the fact that each multiple, regardless of its type and complexity, consists of segments that, from a surface perspective, are primary events (Jakubowicz, 1998; Keydar et al., 1998). We start by picking primary events suspected of being multiple generators on stacked or time-migrated cubes. These picked zero-offset traveltimes are referred to as T_0 surfaces. Given the stacking or migration velocity function along the picked horizon/surface, artificial prestack common-midpoint (CMP) traveltime curves are calculated for each primary event. This procedure can be regarded as destacking (Levin, 2002) and requires no access to prestack data. It is based on the assumption that primary multiple-generator events are hyperbolic. This assumption does not preclude the possibility of the multiples being nonhyperbolic. The limitations introduced by the acquisition pattern are alleviated. If the picked primary horizon covers the entire survey extent, the analytically calculated primaries can be generated everywhere to produce a perfectly ordered (artificial) prestack data set.

These traveltimes are used to predict the prestack traveltimes of the multiples. Each multiple path is determined by a specific multiple condition. A detailed description of the multiple condition and the concept of intermediate points can be found in Keydar et al. (1998); Landa, Keydar et al. (1999) and Landa, Belfer et al. (1999). To complete the poststack prediction, the required multiple times $T_0^{(m)}$ are extracted from the predicted prestack time curves. This method is valid for interbed as well as for surface-related multiple prediction. If the T_0 surfaces are picked on time-migrated images, demigration (Whitcombe, 1994) should be applied before generating the prestack traveltimes. The predicted time surface in this case should be time migrated to appropriately overlay the migrated image. The extension of this algorithm to the 3D case requires modification of the multiple conditions. In the following section we describe the 3D multiple condition for the surface-related and interbed options and the proposed computational algorithm.

3D SURFACE-RELATED MULTIPLES

Figure 1 shows the raypath for a simple surface multiple (peg-leg in this example). If we extend the multiple condition from the 2D to the 3D case, then at the surface point A the emergence angle of the ray starting at the receiver (RCA) must be equal and opposite in sign to the emergence angle of the ray starting at the source (SBA). The multiple time (T_{12} in this case) is given by the simple sum

$$T_{12} = T_{SBA} + T_{RCA}. \quad (1)$$

If we assume that the primary reflection times from the upper reflector ($L1$ in Figure 1) and from the deeper reflector ($L2$ in Figure 2) are available, then an areal search for point A may be performed. In practice, assuming a flat acquisition surface, two emergence angles (one with respect to the x-axis and the other with respect to the y-axis) must be computed from the given primary traveltimes. For a specific source-receiver pair, a point A is searched with the requirement that the emergence angle equality condition will be satisfied in both major directions at A. In addition to the fact that the angle search may become computationally expensive in the 3D case, some

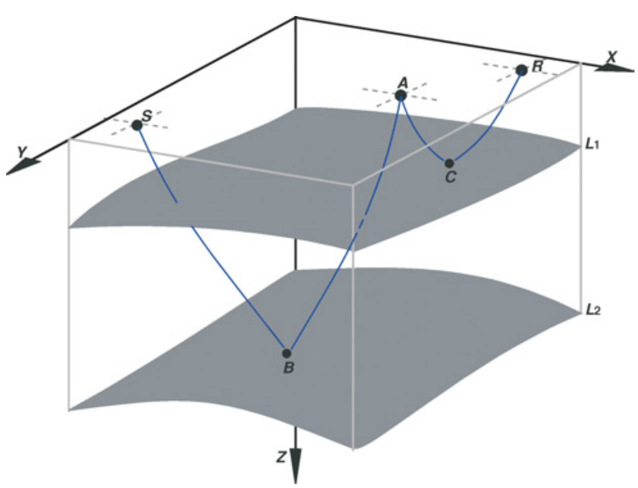

Figure 1. Schematic raypath of a peg-leg multiple for a given shot S and receiver R pair. The surface reflection point A does not have to lie on the line connecting S and R.

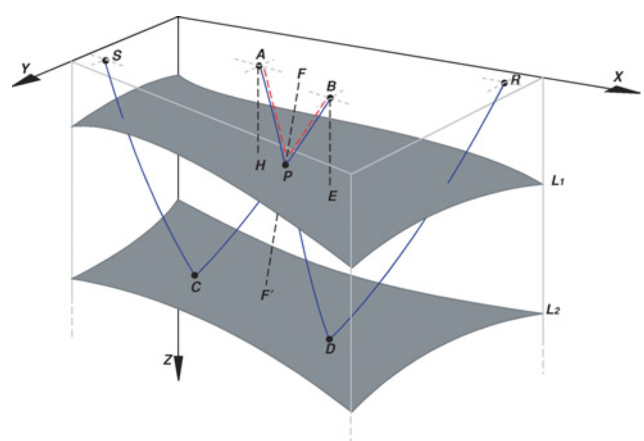

Figure 2. Schematic raypath of an interbed multiple for a given shot S and receiver R pair.

inaccuracy may be introduced because emergence angles must be calculated from the artificial traveltimes (Reshef et al., 2003) or from prestack data (Landa, Keydar, et al., 1999).

An alternative way of finding A can be based on Fermat's principle (Dragoset, 2001). If we assume that the primary reflection paths SBA and RCA (see Figure 1) are minimum timepaths, then given a fixed S and R there exists a point A^* on the surface that minimizes the sum of the primary traveltimes:

$$T_{SBA^*} + T_{RCA^*} = \min_{A \in s_1}(T_{SBA} + T_{RCA}), \qquad (2)$$

where s_1 defines a set of coordinates on the free surface. The minimum traveltime path $SBACR$ (see Figure 1) is therefore the multiple path that also satisfies the angle multiple condition at the surface reflection point A. If the surface-related multiple is of a higher order, then additional primaries will be calculated and more intermediate surface points will be searched.

Our implementation differs from the one presented by Dragoset in the way we extract the primary traveltimes. Rather than calculating the prestack primary traveltimes by forward modeling through a given subsurface model or by picking events on prestack records (Dragoset, 2001), we generate the prestack traveltimes by the abovementioned destacking operation, using T_0 picks on stacked (or time-migrated) cubes and the related stacking (or migration) velocity.

3D INTERBED MULTIPLES

Figure 2 represents the raypath of a simple interbed multiple. As in the 2D case, the multiple time (along the path $SCPDR$) can be calculated from the traveltimes of three different primaries: SCB, RDA, and APB. Since point P, situated on interface $L1$, is the crossing point of the primaries SCB and RDA, we have to show that APB is also a primary reflection. If line FF′ in Figure 2 is normal to $L1$ at P and the path CPD is part of the interbed path, we get from the reflection law the 3D angle's equality:

$$\angle CPF' = -\angle F'PD. \qquad (3)$$

The refraction law at P for path CPB can be written as

$$\frac{\sin(\angle CPF')}{V_+(X_P, Y_P, Z_p)} = \frac{\sin(\angle FPB)}{V_-(X_P, Y_P, Z_P)}. \qquad (4)$$

For DPA is can be written as

$$\frac{\sin(\angle F'PD)}{V_+(X_P, Y_P, Z_P)} = \frac{\sin(\angle FPA)}{V_-(X_P, Y_P, Z_P)}, \qquad (5)$$

where V_+ and V_- are the velocities from below and above $L1$, respectively. By using equality 3 in equations 4 and 5, we get

$$\angle FPA = -\angle FPB, \qquad (6)$$

making APB a simple reflection path. The additional complexity of the interbed case comes from the fact that our approach is based on the analysis of surface-recorded data; point P (positioned on the subsurface reflector $L1$) cannot be determined explicitly. Using a simple multiple condition similar to the one used for the surface-related paths will require the expensive operation of prestack datuming from the surface to $L1$. The implicit determination of P dictates that for a given source-receiver pair, two surface locations (A and B in Figure 2) must be found (Landa, Keydar et al., 1999).

Given the three primary traveltimes and assuming a flat recording surface, the prestack traveltime of the interbed multiple (T_{SR} in Figure 2) can be written as a sum:

$$T_{SR} = T(x_A, y_A, x_B, y_B) = T_{SCB} + T_{RDA} - T_{APB}. \qquad (7)$$

The multiple condition in this case (Keydar et al., 1998) requires that the emergence angle of the reflection SCB be identical to the emergence angle of the reflection from the upper interface APB (see red line in Figure 2). In the same way, the emergence angle of RDA must be equal to the emergence angle of BPA (see red line in Figure 2). If AH and BE are normal to the surface (Figure 2), then from the multiple condition we can define the following angle equalities:

$$\angle PBE = \angle CBE; \quad \angle PAH = \angle DAH. \qquad (8)$$

Although the notation used in equation 8 commonly refers to straight rays, the equality is valid for arbitrary velocity models and for curved raypaths. It means that the emergence angles at the surface of these raypaths are equal.

For simplicity we use the following angle notations:

$$\angle PBE = \beta; \quad \angle CBE = \beta'; \quad \angle PAH = \alpha; \quad \angle DAH = \alpha'. \qquad (9)$$

Next, we use the angle equality (equation 8) to define the ray parameter at A and B:

$$p_{Ax} = \frac{\sin(\alpha_x)}{V_A} = \frac{\sin(\alpha'_x)}{V_A} \Rightarrow \left.\frac{\partial T_{RDA}}{\partial x}\right|_{x=x_A} = \left.\frac{\partial T_{APB}}{\partial x}\right|_{x=x_A}, \qquad (10a)$$

$$p_{Ay} = \frac{\sin(\alpha_y)}{V_A} = \frac{\sin(\alpha'_y)}{V_A} \Rightarrow \left.\frac{\partial T_{RDA}}{\partial y}\right|_{y=y_A} = \left.\frac{\partial T_{APB}}{\partial y}\right|_{y=y_A}, \qquad (10b)$$

$$p_{Bx} = \frac{\sin(\beta_x)}{V_B} = \frac{\sin(\beta'_x)}{V_B} \Rightarrow \left.\frac{\partial T_{SCB}}{\partial x}\right|_{x=x_B} = \left.\frac{\partial T_{APB}}{\partial x}\right|_{x=x_B}, \qquad (10c)$$

$$p_{By} = \frac{\sin(\beta_y)}{V_B} = \frac{\sin(\beta'_y)}{V_B} \Rightarrow \left.\frac{\partial T_{SCB}}{\partial y}\right|_{y=y_B} = \left.\frac{\partial T_{APB}}{\partial y}\right|_{y=y_B}. \qquad (10d)$$

The traveltime along raypath SCB (T_{SCB}) is independent of A, and the traveltime along raypath RDA (T_{RDA}) is independent of B (see Figure 2). Therefore, when we calculate the derivatives of the interbed traveltime at these points, we get

$$\left.\frac{\partial T_{SR}}{\partial x}\right|_{x=x_A} = \left.\frac{\partial(T_{SCB}+T_{RDA}-T_{APB})}{\partial x}\right|_{x=x_A}$$

$$= \left.\frac{\partial T_{RDA}}{\partial x}\right|_{x=x_A} - \left.\frac{\partial T_{APB}}{\partial x}\right|_{x=x_A} = 0, \quad (11a)$$

$$\left.\frac{\partial T_{SR}}{\partial y}\right|_{y=y_A} = \left.\frac{\partial(T_{SCB}+T_{RDA}-T_{APB})}{\partial y}\right|_{y=y_A}$$

$$= \left.\frac{\partial T_{RDA}}{\partial y}\right|_{y=y_A} - \left.\frac{\partial T_{APB}}{\partial y}\right|_{y=y_A} = 0, \quad (11b)$$

$$\left.\frac{\partial T_{SR}}{\partial x}\right|_{x=x_B} = \left.\frac{\partial(T_{SCB}+T_{RDA}-T_{APB})}{\partial x}\right|_{x=x_B}$$

$$= \left.\frac{\partial T_{SCB}}{\partial x}\right|_{x=x_B} - \left.\frac{\partial T_{APB}}{\partial x}\right|_{x=x_B} = 0, \quad (11c)$$

$$\left.\frac{\partial T_{SR}}{\partial y}\right|_{y=y_B} = \left.\frac{\partial(T_{SCB}+T_{RDA}-T_{APB})}{\partial y}\right|_{y=y_B}$$

$$= \left.\frac{\partial T_{SCB}}{\partial y}\right|_{y=y_B} - \left.\frac{\partial T_{APB}}{\partial y}\right|_{y=y_B} = 0. \quad (11d)$$

If we can find two surface points A and B that satisfy the four extremum conditions given by equations 11, the interbed multiple traveltime for the source-receiver pair S–R can be calculated by equation 7. In our implementation, the primary traveltimes are generated analytically, and the required derivatives are calculated after the (artificial) data are sorted to common-source records. For each source-receiver pair, a scan radius is determined; at every surface location within this radius, the extremum conditions are checked until the intermediate points A and B are found. If no solution is found within the predefined area, the scan radius can be increased to include more surface locations.

Different from the surface-related case, Fermat's principle is used here in its general form. The multiple condition is given in terms of the directional traveltime derivatives rather than explicitly using the traveltimes.

EXAMPLES

A 3D synthetic model (Figure 3) is used to test the prediction method. The model covers an area of 12 × 6 km and consists of two layers with a velocity of 1500 m/s above the upper interface, 2000 m/s between the interfaces, and 2500 m/s in the lower half-space. The 3D prestack shot records were calculated over this model by kinematic ray

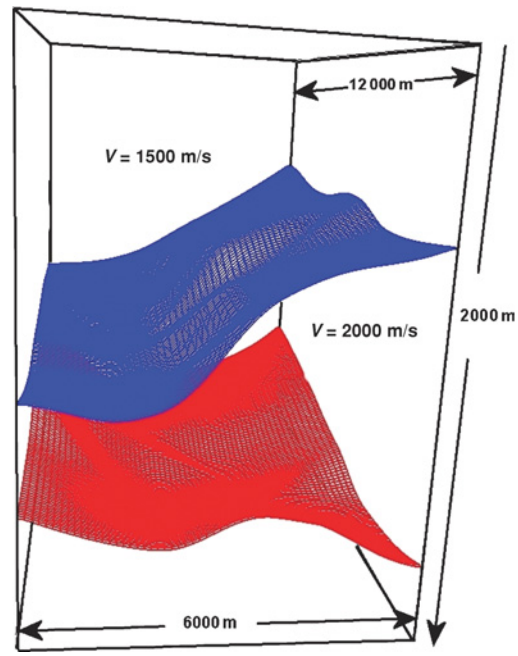

Figure 3. A 3D synthetic model. The horizontal size is 12 × 6 km; 1500 m/s is the velocity V above the first interface, 2000 m/s is the velocity between the first and second interfaces, and 2500 m/s is the velocity in the lower half-space.

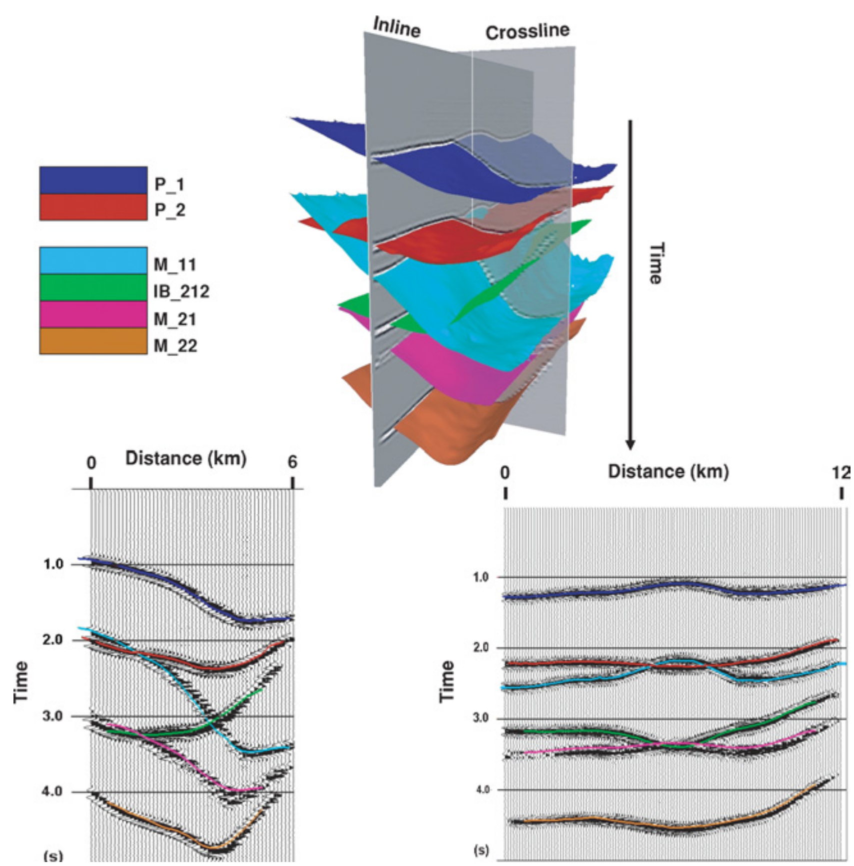

Figure 4. Multiple prediction, synthetic example. (Top) A 3D view of the two primaries and four predicted multiples. (Bottom) Inline and crossline displays from the central part of the cube. The picked primaries and the predicted multiples are color coded according to the table on the upper left.

tracing. A stacked cube was obtained after sorting the synthetic data to CMPs, picking a stacking velocity, applying NMO correction, and stacking the corrected gathers.

Six events were modeled in this example: two primaries and four multiples. The primaries (marked P_1 and P_2) were reflected from the two interfaces. The multiples were the two first-order surface multiples from the first and second interface (M_11 and M_22), a peg-leg from the second interface (M_21), and an interbed multiple (IB_212). The two primaries were picked on the stacked cube, and a stacking velocity was extracted along each one of the two picked T_0 horizons.

The 3D prestack traveltimes were generated, using hyperbolic approximation, for each CMP location along the two horizons. Four multiples were predicted using these prestack primary traveltimes. The results of the prediction are summarized in Figure 4. The surface-related multiples were predicted using Fermat's principle and show a good match to the modeled ones. For the prediction of the interbed multiple, we used the extremum conditions (see equations 11) to calculate the predicted surface. As in the case of the surface-related multiples, the match with the modeled data is good.

In many situations where 2D lines are processed and interpreted, the truly 3D multiples are impossible to handle correctly. If specific multiple-generating horizons can be mapped (from a multi-2D data set), our method can predict the 3D multiples, which can later be superimposed on the 2D line for more accurate interpretation and processing quality control (QC). We tested this procedure on a set of 2D marine lines; the result is presented in Figure 5. A portion of a peg-leg (see arrow on Figure 5b) was accurately predicted by our method.

Multiple prediction without the need to access prestack data can be a very powerful tool for interpretating and processing QC. The next real data example demonstrates how the method is used as a QC tool to check if significant multiple energy remains in the processed cube. Figure 6b shows a small 3D marine stack on which two primaries, suspected to be multiple generators, are picked (P_1 and P_2 in Figure 6b). A full surface multiple from the second interface (M_22), a peg-leg from the first and second interfaces (M_21), and an interbed multiple (IB_212) were predicted. An inline from the center of the cube (Figure 6a) shows that these multiples are not present in the data.

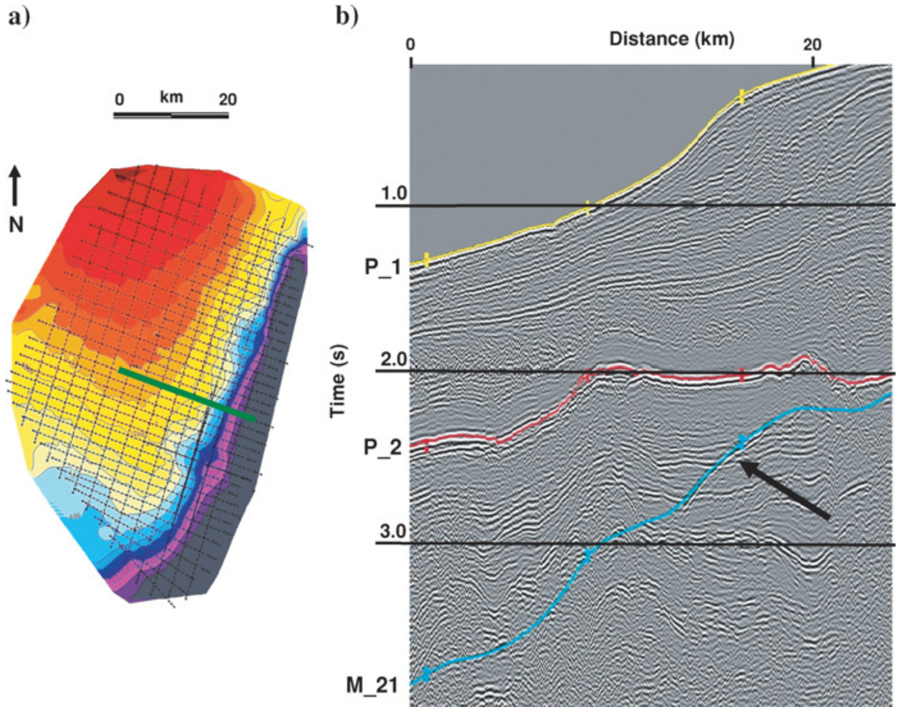

Figure 5. Three-dimensional multiple prediction from a 2D survey. (a) Survey map showing the 2D acquisition pattern overlain by a T_0 map of the first primary reflector. (b) Portion of a 2D line [location marked by the green line on (a)], showing two picked primaries (P_1 and P_2) and a predicted peg-leg (M_21). The arrow points to significant multiple energy in the section.

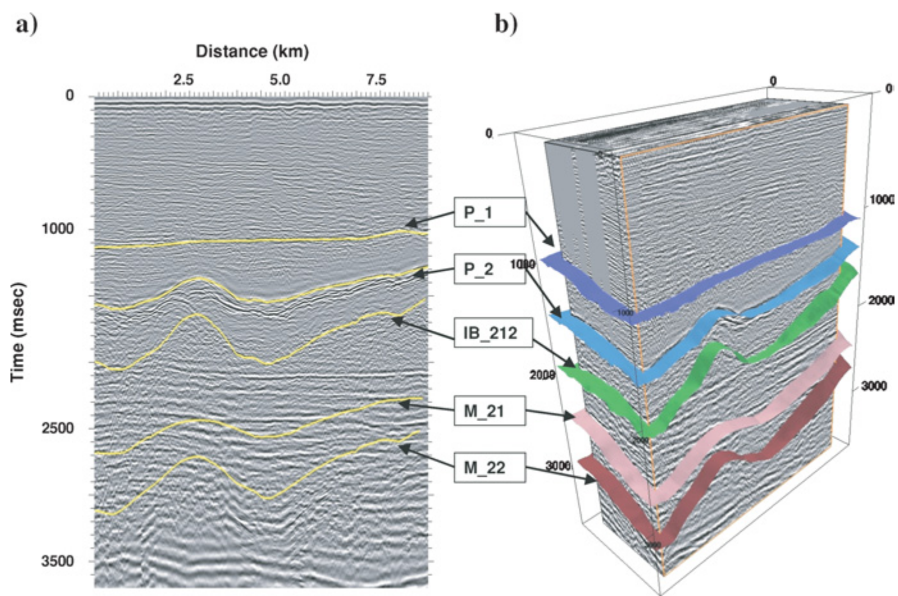

Figure 6. Marine data QC. (a) Two primaries (P_1 and P_2), two surface-related multiples (M_21 and M_22), and an interbed multiple (IB_212) are shown on top of a central inline taken from a stacked cube (b). No coherent events exist along the predicted multiples' horizons.

DISCUSSION AND CONCLUSIONS

A kinematic method for predicting 3D multiples, both surface related and interbed in the time domain has been presented. The method can be classified as a poststack technique that does not require access of prestack data and is highly suitable for interpretive work.

As opposed to earlier 2D studies, the multiple conditions are calculated efficiently by implementing Fermat's principle, thus avoiding the need to estimate emergence angles of primary events from prestack traveltimes. The computational cost of our method is directly related to the order and type of the predicted multiple. Simple surface-related multiples (for example, first- and second-order water-bottom multiples or peg-legs) require a couple of minutes of computing time using state-of-the-art interpretation stations. For multiples of the same order, the cost of predicting interbed multiples is at least twice the cost of predicting the surface-related ones. In general, the computational cost is dictated by the number of surface intermediate points needed to satisfy the specific multiple condition.

The major limitation of our method comes from the assumption that our primaries are hyperbolic. As long as this assumption holds, our prediction method does not have dip limitations. If this is not the case and the prestack primary traveltimes are nonhyperbolic, the predicted multiples may be mispositioned. Since our procedure is implemented in the time domain, we find this assumption reasonable and practical.

The computational algorithm can be used to predict 3D multiples in the depth domain. The difference is in the way the prestack primary traveltimes should be calculated. After picking suspected multiple-generating reflectors on the depth section and by assuming that an interval velocity model is available, kinematic forward modeling (upward only) must be executed to calculate the prestack traveltimes. Our algorithm can be applied to predict the multiples in the time domain. To finalize the process, each predicted multiple surface should be map migrated from the time domain to the depth domain.

Although this study was limited to predicting poststack multiples, our method provides 3D estimation of the multiples in the prestack domain. Since the method is insensitive to acquisition geometry, we believe the possibility to use the method for multiple suppression warrants further investigation.

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