Validating the velocity model: the Hamburg Score

The late Vladimir Glogovsky,[1] with co-authors Evgeny Landa,[2*] Sergey Langman[1] and Tijmen Jan Moser[3] discuss the fundamental properties of the kinematic inversion problem which continue to challenge geophysicists. Recognizing the 'ill-posed' character of the inverse problem, they argue that it does not have a solution in a strictly defined sense. Acknowledging the importance and limitations of a priori information, they go on to present an approach to a 'well-posed' solution which relies on constructing an extension of the kinematic inverse problem by making additional assumptions and validating them.

The German mathematician Oskar Perron is well-known for a paradox which highlights the danger of assuming that a solution to a problem exists. The paradox runs as follows:

Suppose the largest natural number is N. Then, if $N>1$ we have $N^2 > N$ contradicting the definition. Hence, the largest natural number is equal to 1! Clearly, we arrive at this absurd conclusion because we assumed that the largest natural number exists. When dealing with problems of a real physical nature and with real objects, the analogies with Perron's paradox become even deeper. Quantitative description of such objects is always done in a certain model, and the results obtained have a practical meaning only if the solution is correct, i.e., the problem is well-posed. The mathematical term 'well-posed problem' stems from a definition given by Hadamard (1902). He postulated that mathematical models of physical phenomena should have properties where:

1. A solution exists.
2. The solution is unique.
3. The solution depends continuously on the data.

Verification of these conditions is often not a trivial task because the solutions obtained may not be a priori as absurd as in Perron's paradox. An apparently reasonable result can mistakenly create an illusion that the problem is solved. Problems that are not well-posed in the sense of Hadamard are termed 'ill-posed'. In Hadamard's opinion, an ill-posed problem has no physical sense. It is generally agreed today that many ill-posed problems have so-called well-posed extensions which are very meaningful. These extensions introduce a priori assumptions about the unknowns. If a problem is well-posed, then it stands a good chance of finding a solution on a computer using a stable algorithm. If it is not well-posed, it needs to be re-formulated before numerical treatment. Typically this involves the inclusion of additional assumptions, such as smoothness of the solution. This process is known as regularization (Tikhonov, 1963).

It is well-known that geophysical inverse problems are as a rule ill-posed. Nevertheless, we still need to try to develop methods for extracting information about the subsurface from geophysical data. For a long time such methods have been merely heuristic. Backus (1970) made the first systematic exploration of the mathematical structure of inverse problems. Tarantola (1987) in his book took the view that the most general formulation of inverse problems can be obtained by using the language of probability calculus and

In memory of Vladimir Glogovsky

This legend about the Hamburg score was told to us by Vladimir many years ago....

According to the legend popular among Russian scientists, the method to judge their own accomplishments, and those of their peers, was known as the 'Hamburg Score'. This term was coined in 1928 by the famous Russian literary critic Viktor Shklovsky with reference to wrestlers' competition. Then as now, wrestling was more of a show than sport …

'All wrestlers cheat in performance and allow themselves to lose a fight at the behest of the organizers. But once a year wrestlers gather in Hamburg and fight each other in private contests without the public. It is a long, hard, ugly competition. But this is the only way that they can reveal their real class.'

Vladimir's life was a real Hamburg Score…

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the popular Bayesian approach (see Ulrych et al., 2001) for a tutorial). In his opinion this is the only approach which allows the analysis of concepts such as errors and resolution in the solution with a convenient degree of generality. This approach is based on a purely statistical view of seismic data and presumes knowledge of the statistical properties of the model before using the data (a priori information), as well as the statistical properties of the data. According to the Bayesian approach, the data is used in inversion to constrain the a priori model, and not the opposite as when the inversion is constructed from the data and the a priori model serves as a constraint. In a further commentary, Tarantola (2006) presents an even more extreme view in which 'observations cannot produce models, they can only falsify models'. In this view, derived from Popper's philosophy, the inverse problem is to be solved by generating large numbers of random models, discarding those that are disqualified by the data and keeping the others.

However, while Bayes's theorem is in itself indisputable, the main problem with the Bayesian or probabilistic approach is that in practice our a priori information of the Earth's interior is very poor. In particular, the probability density functions attached to data and model parameters are usually unknown (Scales and Snieder, 1997). Most of the theoretical models are based on the Gaussian assumption of noise. Thus, solutions are limited by the well known least-squares method. All this limits the effectiveness of statistical procedures. By contrast, a deterministic approach to inversion may be formally characterized by uniform probability density functions for the errors in the model and the data, but this still has the problem of how to incorporate a priori information.

A case in point is the inverse kinematic problem using a traveltime inversion approach (Dix transformation, coherency inversion, traveltime tomography, etc). Here, arrival times of reflected waves, picked or estimated in one way or another on prestack gathers, serve as input data. They should be picked or estimated (in one way or another) on prestack gathers. Numerous published results have shown that in complex structural situations velocity model building on the basis of observed data becomes the most important and difficult practical problem. Some of the conclusions of these studies are: a) the process of converting traveltimes to interval velocities is unstable for layers with lateral velocity variations (Bickel, 1990; Stork and Clayton, 1992); b) seismic traveltime data exhibit ambiguities that prevent the resolution of time anomalies into structure and interval velocity (Tieman, 1994); and c) only the long wavelengths of the subsurface velocity variations can be recovered with confidence (Kosloff and Sudman, 2002). In other words, traveltime inversion methods all create velocity models that can correctly model the recorded traveltimes, but which are geologically unrealistic or have different geological implications.

It is important to emphasize that these conclusions are not dependent upon the particular choice of algorithms (misfit norm, forward modelling, misfit minimization technique). Instead, they are a feature of the subsurface and its seismic response, and as such a fundamental problem in reflection seismology. In principle, to create a more feasible model it is necessary to add a regularization term to the objective function. In theory, this can be achieved by proper incorporation of different types of a priori information. Still, it is of great importance to understand and to assess which information comes from the data, which from prior information and which is just from the geoscientist's experience in the art of velocity model building. Quantifying this is a big challenge as there are too many uncertainties (data errors, prior probability distributions, etc) to do it rigorously, e.g., via a Bayesian framework. Incorporating all this quantitative information into the objective function is also not a trivial task (Clapp, Biondo and Claerbout, 2004).

Resolving the problem of velocity model construction has become an end in itself, and the search for a solution has somehow overshadowed the more important problem of getting the correct depth image. This depends on the estimation of a velocity model, the inversion algorithm, and the chosen class of models (number of reflection interfaces, parameterization for interfaces and velocities, etc). The latter is usually selected from a priori geological information and/or interpretation of stacked time sections. If different models provide the same results with reasonable accuracy, there should be a way either to select the true model or to admit that a unique depth-velocity model cannot be obtained – even when the required characteristics of the wavefield have been accurately modelled.

In fact, the most difficult point of the inverse seismic problem is how to distinguish between two 'correct' models. (It is not too difficult to distinguish a correct model from an incorrect one). This problem requires special techniques, and migration (including prestack) by itself cannot resolve it. Although the problem of uniqueness and stability of the inverse kinematic problem is a hotly debated issue (see the reference list), most of the published papers discuss this problem assuming that the model parameterization (number of layers, spline description of the reflectors and velocities, etc) is known and correct. At the same time the question of model adequacy (parameterization) is discussed much less (we mention only a few publications on this issue: Landa et al., 1998, Glogovsky et al., 2002). In practice the problem is much worse and such issues as the effect of the offset range, type of regularization, and concrete inversion algorithm are largely irrelevant for practical inverse problems because they typically do not have solution (if by solution we mean an Earth model).

In this paper we will not propose or describe a new inversion algorithm or scheme. We will not compare different methods and algorithms. Rather we will discuss the following questions: what solution of the inverse kinematic problem means, and which properties of the inversion is it important to understand in order to obtain a geologically meaningful solution? The paper has a conceptual character and is therefore not structured in sections dealing with theory and numerical examples; rather, the examples will continuously support the argumentation. In the first section 'Choosing the model' we discuss the problem of finding an adequate model parameterization. The section 'Stability of the inverse kinematic problem' deals with the non-uniqueness of solutions and their implications. Finally, in 'What to do?' we discuss the merits of a priori information and the possibility of well-posed extensions to an ill-posed problem.

Choosing the model

Most modern methods for kinematic inversion use the so-called layered model description of the subsurface. We normally assume that the Earth contains n layers and that the velocity in each layer and the interfaces between the layers can be adequately described by a certain class of functions which are characterized by a set of parameters. The solution of an inverse problem in this case means finding parameters which, when used in forward traveltime computations, minimize the differences between the observed and the calculated traveltimes for the main reflection events.

The selection of the model type for each region (or profile in the case of 2D case) is usually done according to geological information and a time cube/section. Fig. 1 shows a typical time section. Strong continuous time horizons are usually chosen and interpreted (four green solid curves in the figure) and prestack arrival traveltimes for these horizons serve as input information for kinematic inversion. There are many coherent events between the first and the second horizons but it is probably not a good idea to include all of them in the inversion. Velocity determination in thin layers is unreliable and error accumulation will reduce the precision of the velocity estimation in the deeper part. The same can be concluded regarding the target interval between the third and the fourth horizons.

Here we want to note that the problems raised are relevant to both complex and simple structural situations. We are mostly interested not in the global structural behaviour of the reflection interfaces but rather in their local features (structures, pinch-outs, faults etc). The difficulty of velocity estimation is proportional to the seismic data complexity, and inversely proportional to the dimensions of the structure which may have an exploration interest. We can express this in a symbolic linguistic equation:

$$\frac{\textit{Seismic data complexity}}{\textit{Dimensions of local structure}} = \textit{Complexity of velocity estimation}$$

The right hand side of this equation can be taken as a universal constant for all exploration regions.

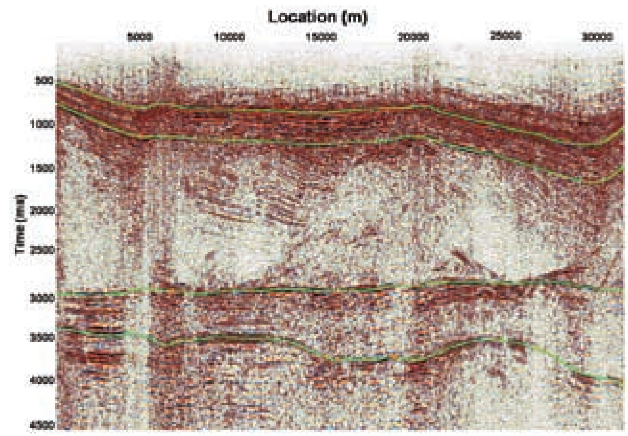

Figure 1 *An interpreted stacked section.*

A subsurface model can be parameterized in different ways. In many inversion algorithms, polynomial or spline parameterizations for reflection interfaces and velocities are used. In several algorithms (like the one described in this paper) a local constant velocity approach is used. In this case we assume that the velocity in a particular layer is constant at least in a certain horizontal interval comparable with the acquisition spread length and that the reflection interface can be determined by a simple map migration procedure for normal rays. Note, however, that our discussions and conclusions have a general character and do not depend upon a specific type of model description and parameterization.

Let us assume that the interfaces are parameterized by functions $h_i(x,y,z,\vec{\alpha})$, where i is the reflector number, x,y,z are Cartesian coordinates and $\vec{\alpha}$ is a vector of unknown parameters. The interval velocities between the reflectors are described by functions $v_j(x,y,z,\vec{\beta})$, where j is the layer number, and $\vec{\beta}$ is a vector of unknown parameters. These functions belong to given classes $h_i \in \langle H \rangle$, and $v_j \in \langle V \rangle$. The choice of the classes H and V is usually dictated by the type of modelling procedure that is available. If we are tracing rays, we need ray-tracing friendly models (e.g., smooth velocity and interface geometry functions). Similarly, finite-difference techniques work on grid models. This choice has important implications for the behaviour, solvability, and well-posedness of the inverse problem.

After choosing the functions from these classes we try to find the parameters $\vec{\alpha}$ and $\vec{\beta}$, in such a way that reflection traveltimes calculated by tracing rays through the model coincide with the observed traveltimes. For any given set of velocity function $v_i(x,y,z,\vec{\alpha})$ and reflection interface $h_j(x,y,z,\vec{\beta})$ a traveltime surface can be computed by tracing rays through the model. It is clear that the calculated traveltimes can coincide with the observed ones only if the classes $\langle H \rangle$ and $\langle V \rangle$ contain functions which adequately describe the reflection interfaces and velocities within the

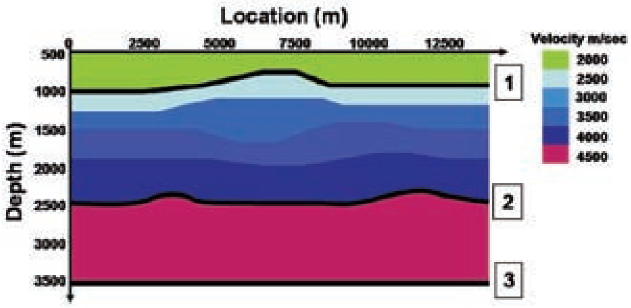

Figure 2*a Synthetic velocity-depth model.*

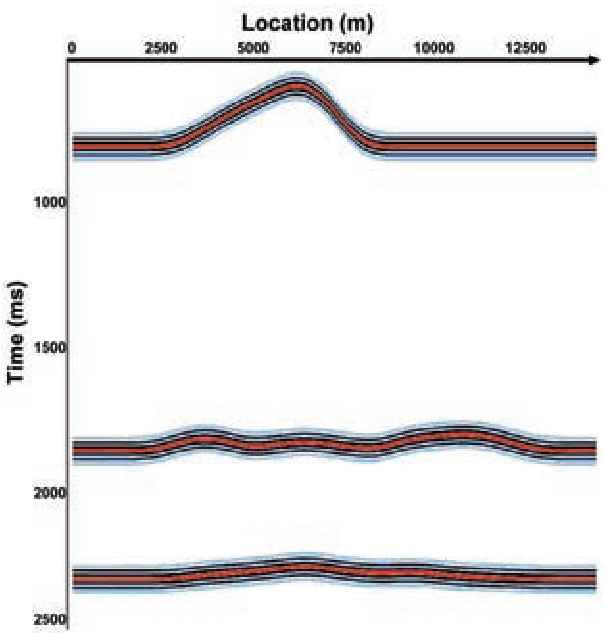

Figure 2*b Synthetic zero-offset section.*

true Earth. Thus if $v(x,y,z,\vec{\alpha}) \notin \langle V \rangle$, or $h(x,y,z,\vec{\beta}) \notin \langle H \rangle$ then there are no parameters $\vec{\alpha}$ and/or $\vec{\beta}$ for which the calculated and the observed traveltimes coincide.

Now we come to our first important conclusion. The main problem of inversion is not, as it is often perceived, to find the best parameters $\vec{\alpha}$ and $\vec{\beta}$ - given classes H and V. This problem is relatively easy to solve, and we have the tools for it. Instead, the most important problem of inversion is where to look for $\vec{\alpha}$ and $\vec{\beta}$. What are the appropriate classes H and V? Is the subsurface really 1D-, 2D- or 3D-layered? Can the interval velocities really be described adequately by constant, linear, or spline functions, and the interfaces by, e.g., piecewise splines? These questions are typically impossible to answer conclusively, and they are usually ignored. Because we never know for sure to which classes H and V the true functions h_i and v_i belong, we can say that the inverse kinematic problem does not have an exact solution. Thus, kinematic inversion is an ill-posed problem. Practically it means that we should decide or define what we will consider as a solution of the

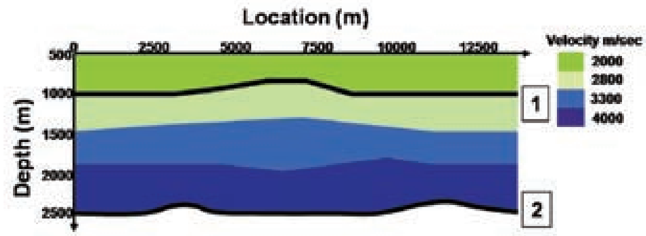

Figure 3 *Inversion result for the second layer. The misfit objective function is less than 1 ms.*

inverse kinematic problem. Usually we choose a solution (a parameters set $\vec{\alpha}$ and $\vec{\beta}$ for given functions classes H and V) which *minimizes* the difference between calculated and observed traveltimes..

In reality, the statistical properties of errors in the observed traveltimes are not known and it is difficult to estimate them reliably. It means that we can use the best traveltime fit as a criterion for the inversion, but only an *acceptable* difference between calculated and observed data (defined in some sense). How much the solution allows for reconstruction of important structural characteristic of the subsurface and what the requirements are for the model obtained with these characteristics? These are the most difficult and still open questions. At the same time the importance of these characteristics is a factor external to the inversion problem, which is usually defined by geologists and generally depends on the exploration region.

Stability of the inverse kinematic problem

Let us consider the stability of the inverse problem for the solution defined above. If we model a situation similar to the one shown on the seismic section displayed in Fig. 1., then Fig. 2 shows a velocity-depth model containing three reflection interfaces (black solid lines in Fig. 2 a) with constant velocities in the first and third layers. To simulate the velocity inhomogeneity of the second layer we use four fictitious layers with constant velocities and non-reflecting boundaries. Figure 2 b) shows the corresponding zero-offset section.

We assume that we already successfully estimated velocity and interface geometry parameters for the first layer. Figs. 3 and 4 illustrate results of two inversion runs for the second layer. In one case (Fig. 3) we fixed the correct position of the second reflection interface and inverted only for velocity, using an inadequate velocity parameterization (simulating the inhomogeneity between the first and second reflectors by three, instead of four, intermediate homogeneous layers). In the other case (Fig. 4), we inverted for both reflector position and velocity.

It is not important for the purposes of this paper which specific inversion algorithm was used to estimate velocity-depth model. What is important is that the

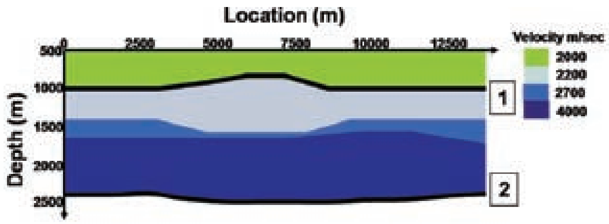

***Figure 4** Another inversion result for the second layer. The misfit objective function is less than 1 ms.*

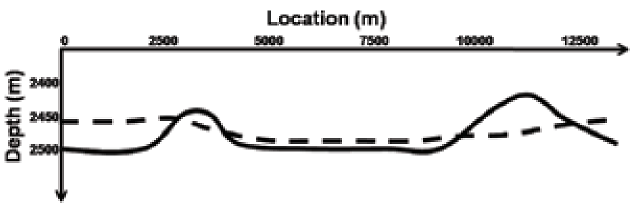

***Figure 5** A zoomed version of the resulting depth models for the second layer. The two interfaces are different from geological point of view: one model (dashed line) contains a monocline on the right hand side, and the other model (solid line) contains an anticline structure with amplitude of about 100 m.*

prestack traveltimes calculated for *both* of these estimated models fit the observed traveltimes for the second reflector with a precision of one millisecond. Fig. 5 shows a zoomed version of the resulting depth position for two models. The two solutions are different from each other in a geological sense: one model (dashed line) contains a monocline at the right hand side, and the other model (solid line) contains an anticline structure with depth variations of about 100 m. In this case a priori information cannot be used to conclusively resolve the problem of non-uniqueness. If a vertical well was drilled at the horizontal location x=3750 m (top of the anticline structure), it would confirm the depth and velocity obtained in both inversion result. As indicated by the figure, it is a wrong conclusion because the coincidence between the estimated and supposedly true models is local around the well only. Note that the same conclusion applies if we had drilled wells at the horizontal locations x=3000 m, 10,000 m, and 12,500 m. Of course, at other locations there would be a mis-tie, but that does not refute the general fact that a correct tie cannot be used to distinguish between models.

This experiment illustrates the well-known fact that kinematic inversion based on the criterion of minimizing the difference between calculated and observed reflection traveltimes may be unstable and can produce multiple acceptable solutions. We quote here from S. Treitel's (1989) paper: '...we learn that a good fit is a necessary but by no means sufficient condition for success. By itself, a good fit does not guarantee that an inversion is correct. This occurs, in my opinion, more often than we would like to think'. Although it is known that incomplete data result in a null space of models which all result in the same best fit (Jackson, 1972, Jannane et al., 1989), many modern inversion schemes and algorithms are based exclusively on the fitting criterion, deceptively suggesting a unique solution.

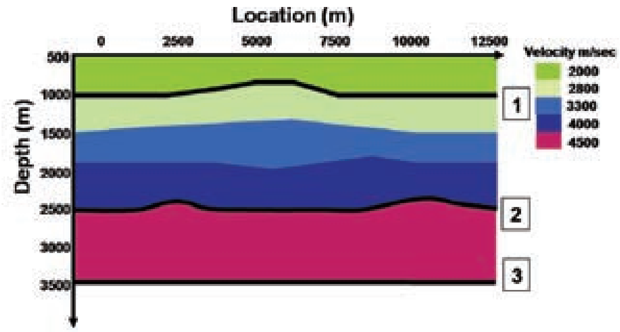

***Figure 6** Results of the inversion for the deepest layer. The misfit objective function is small (about 2 ms).*

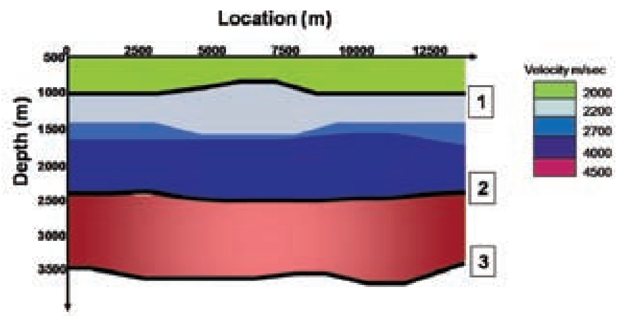

***Figure 7** Results of the inversion for the deepest layer. The overburden model in this case was incorrect. The misfit objective function is small (about 2 ms).*

Let us continue the inversion process and estimate parameters for the deepest layer (third reflection interface on the time section shown in Fig. 2). Figs. 6 and 7 show two results of the inversion. In one case (Fig. 6), where the overburden reflector was correctly positioned, we obtained correct values for the reflection interface and velocity. In the other case, when the overburden model was wrong (Fig. 7), inversion results for the deepest layer are wrong too. Note that the misfit objective function in both inversion runs is very small (about 2 ms), which means that the computed traveltimes fit the observed data optimally. In the second case, to reach an acceptable level of the objective misfit function, we allowed lateral velocity variation whereas in the true model the velocity is constant. It is almost obvious that, if for the second inversion run we had choose to set the correct velocity and/or depth position in the last layer, the misfit function would have been larger due to a wrong overburden found at the previous inversion steps. These experiments clearly demonstrate that:

1. Inversion based on the best fit of observed and calculated reflection traveltimes may lead to construction of several subsurface models with significantly different geological meaning, all of which fit the observed data equally well.
2. An overburden model constructed by the best fit of observed and calculated reflection traveltimes does not guarantee a correct solution for the deeper part of the model.

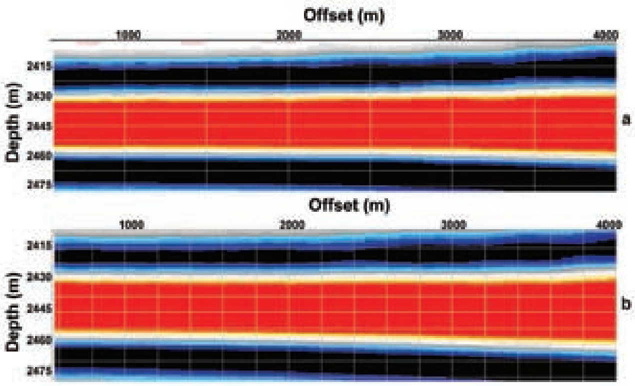

Figure 8 *Two typical common image gathers (CIGs) for the third (deepest) reflector obtained for the two different velocity models shown in Figs. 6 and 7.*

3. Refinement of the model parameterization may lead to a better fit of the calculated and observed traveltimes but does not guarantee construction of a better subsurface model.

Can prestack depth migration (PSDM) and common image gathers (CIGs) help us to choose a correct velocity model and distinguish between realistic and unrealistic models? It is obvious that if the observed and computed reflection traveltimes differ by no more than 1–2 ms, then the CIGs will be perfectly horizontal at least for the main reflection events. Fig. 8 illustrates typical CIGs for the third reflector obtained for the two different velocity models shown in Figs. 6 and 7. Note that in both models the CIGs are flat for all horizontal locations, because of the correspondence between calculated and observed arrival traveltimes, therefore we show only one CIG per model. Figures 9 a) and b) show the results of PSDM for the two last interfaces for these models. Migration results look very different but both satisfy the fitting criterion and produce flat CIGs. Thus the use of CIGs and PSDM does not help in this case to choose a supposedly correct solution.

What to do?

The answer to this question is to develop a theory of kinematic seismic inversion. Today a large number of semi-heuristic algorithms and strategies exist for kinematic inversion. They contain many recipes and sometimes a hint of which solution to accept, but they do not solve the inverse problem conclusively. The examples shown above have a general character and are not illustrating a rare, particular situation. Conclusions obtained from these examples do not characterize concrete algorithms. Rather, they illustrate fundamental properties of the inversion problem and can serve as counter-examples to common practices and assumptions. (Recall that in mathematics a counter-example is an exception to a proposed general rule, i.e., a specific instance of the falsity of the 'for all' statement).

Without basic definitions, proofs, and solutions of many internal contradictions, we cannot have meaningful improvement in velocity-depth model construction. As cited in Scales and Snieder (1997) '...in order to treat inverse problems in ways that are different from current practice requires significant theoretical and numerical advances.' Bayesian inversion which in recent years has gained a strong popularity in its application to geophysical inverse problems in principle could give answers to these questions.

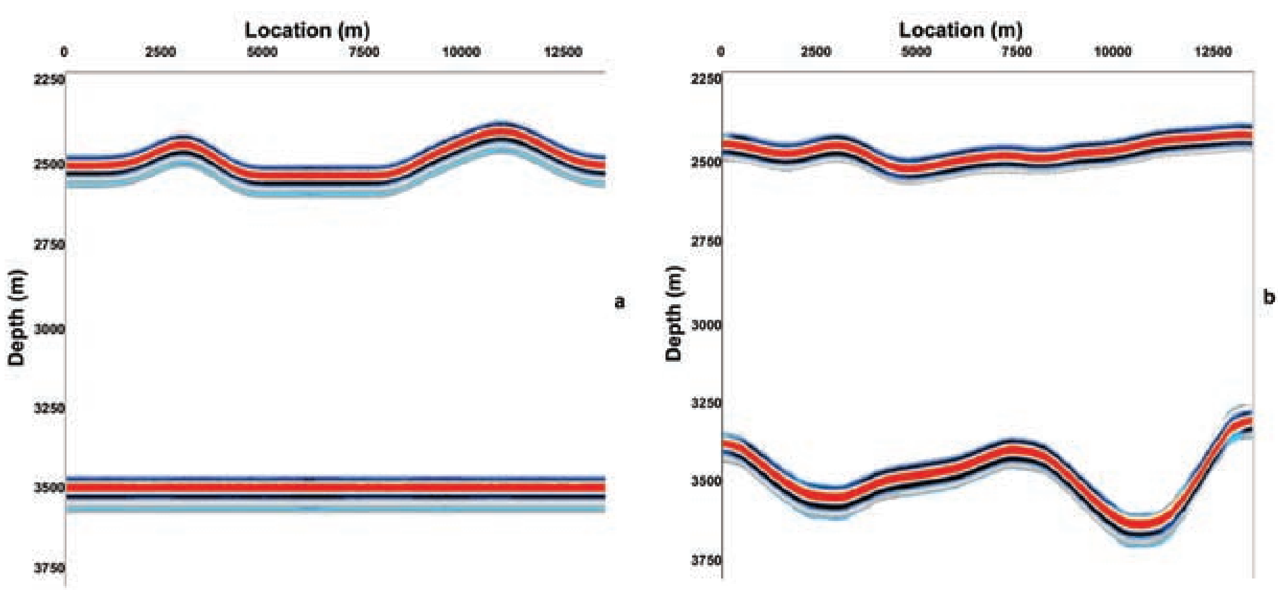

Figure 9 *Results of PSDM for the two last interfaces for two models shown in: a) Figures 6, and b) Figure 7. Migration results look very different but both satisfy the fitting criterion and produce flat CIGs.*

It provides a framework for combining the a priori model information with the information contained in the data to arrive at the a posteriori model distribution. But, in practice, it is extremely difficult to use Bayes's theorem to do realistic inverse problems (Scales and Snieder, 1997). This happens, in our opinion, because very often the mathematical representations of the a priori model information as well as the statistical description of the data statistic are not justified by the observations. Although the importance of a priori information is well understood and widely discussed (Jannaud and Delprat-Jannaud, 1994, Tarantola, 1987), until now there is no complete theory on how to optimally and critically use this information. The limitations of a priori information need to be well understood.

Let us summarize the general kinematic inversion scheme as follows. We have a space of velocity-depth models $\langle M\rangle$, time space $\langle T\rangle$ and an operator φ such that $\varphi(m)=t$ when $m\in M$ *and* $t\in\langle T\rangle$. If $\hat{t}$ is the best approximation for *t*, does it mean that $\varphi^{-1}(\hat{t})$ is the best approximation for *m*? The answer is: not necessarily! Mathematical proof for this is outside the scope of this paper, but is related, among other things, to the operator φ being one-to-one, which is seldom the case in practice.

The above examples illustrate that the best approximation of traveltimes does not guarantee the best approximation of the subsurface model. In other words, the statement that a model estimated using a criterion of the best traveltime fitting is the correct subsurface model is, in a general sense, meaningless. What can we do? We believe that the seismic method has the potential to answer (at least partly) the questions raised. In Hadamard's sense, we may try to reformulate the ill-posed problem in a way that it becomes well-posed, that is, has a solution which is unique and stable. This can be done by adding an assumption and validating this assumption against the observations – in fact, the validation is the most crucial step in inversion.

One of the interesting possibilities is related to the use of the layered model description and locally homogeneous assumption. Let us assume that the reflection interface is locally planar and the velocity in the layer under investigation is constant in a vicinity of the reflection point. In this case, the inverse problem becomes well-posed, can be solved with respect to the velocity, and is over-determined with respect to boundary conditions: for a single unknown velocity parameter, we have two boundary conditions using Snell's law for transmission at the top or reflection at the bottom of the layer. In this case the second (free) boundary condition can be used to check if the input data satisfy the assumption of local homogeneity of the layer (Glogovsky and Gogonenkov, 1987, Glogovsky et al., 2002). The inverse problem in this case can be solved in two different ways. Fig. 10 shows the scheme for velocity

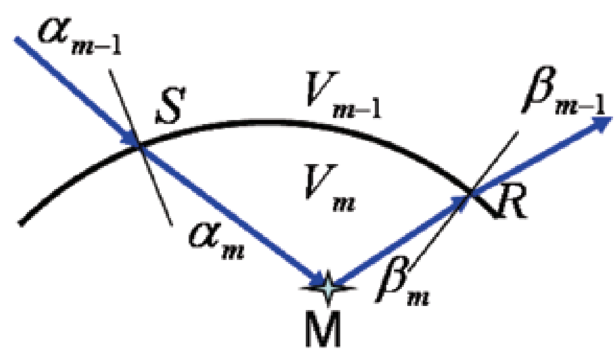

Figure 10 *Inversion scheme to compute velocity V_m using Snell's law at the top of the layer at reflection point M, S and R are the positions of the downward and upward branches of the ray. α_{m-1} and β_{m-1} are the incident and the refraction angles respectively at points S and R of the intersection of the rays with the (m-1)-th interface.*

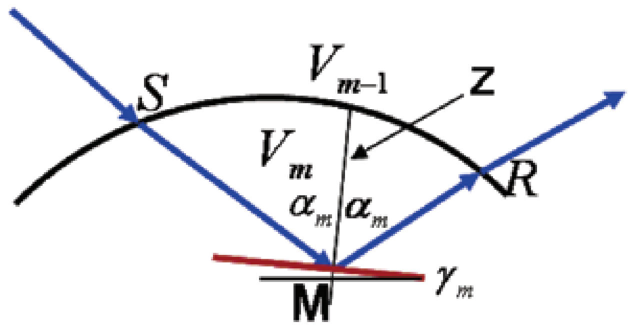

Figure 11 *Inversion scheme to compute velocity V_m using Snell's law at the bottom of the layer at reflection point M. In this case the traveltime of the wave from the source to receiver is a function of the source-receiver coordinates and the three parameters of the layer, namely, velocity V_m, depth Z, and dip of the reflector γ. Three source-receiver pairs are enough to determine the unknown parameters.*

estimation using Snell's low at the top of the layer. We assume that we already know the overburden velocity V_{m-1}. Using Snell's law we find the positions *S* and *R* of both the downward and upward branches of the ray in layers with numbers *k=1,...,m-1*, according to the values of $\partial t_k/\partial x_s$ and $\partial t_k/\partial x_r$, where t_k is the reflection traveltimes and x_s, x_r are source and receiver coordinates respectively. We also determine the incidence angle α_{m-1} and the refraction angle β_{m-1} at points *S* and *R* of the intersection of the rays with the *(m-1)-th* interface. The problem has then been reduced to determining the velocity V_m in the *m-th* layer, according to the residual traveltimes in the *m-th* layer δt_m (where δt_m is the difference between the reflection traveltimes $t_m(x_s,x_r)$ and $t_{m-1}(x_s,x_r)$) and angles α_{m-1} and β_{m-1}. Another way to compute velocity V_m is to use Snell's law at the bottom of the layer at reflection point *M* (Fig. 11). In this case the traveltime of the wave from the source to receiver is a function of the source-receiver coordinates and the three parameters of the layer, namely, velocity V_m, depth Z, and dip of the reflector γ (see the figure). Thus, three source-receiver pairs are enough to determine the unknown parameters.

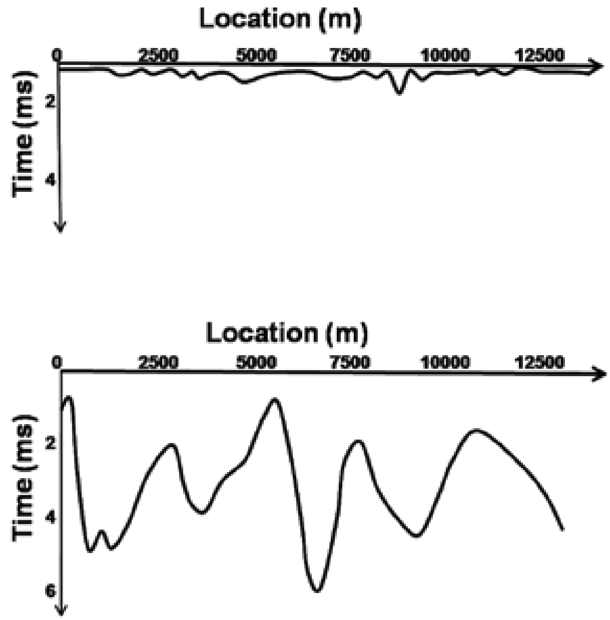

Figure 12 *The adequacy criterion for two models for the deepest layer: (a) for the model shown in Fig. 6 and (b) for the model shown in Fig. 7. The criterion indicates that the model shown in Fig.6 better satisfies the local homogeneity assumption and thus the results of this inversion are more reliable than the results shown in Fig. 7.*

The difference between two inversion results obtained by using Snell's law at the top and at the bottom of a layer can serve as a criterion for a local homogeneity assumption. Small values of this criterion indicate the validity of our assumption on a locally homogeneous layer and a high reliability of the obtained inversion results. Fig.12 illustrates the behaviour of this criterion for two models of the deepest layer obtained in two inversion runs: one shown in Fig.6 and the second shown in Fig. 7. The criterion clearly indicates that the model shown in Fig. 6 better satisfies the local homogeneity assumption and thus the results of this inversion are more reliable than the results shown in Fig.7. In other inversion schemes, when the estimated velocity is not assumed to be constant (typically in different tomographic schemes), we may not have enough boundary conditions to check the adequacy of the model description and the input data.

Discussion and conclusions

Our paper is addressed to geophysicists working in the area of inverse problem. We have tried to formulate a number of fundamental questions which in our view should be addressed in order to make the field of geophysical inverse problems a mature science, rather than a set of recipes (and sometimes questionable recipes).

We have demonstrated that for an arbitrary velocity-depth subsurface the inverse kinematic problem might not have a solution in the strictly defined sense. The reason for this is that it is impossible to rigorously verify whether a certain parameterization of the subsurface model is adequate for the problem at hand. Thus, the inverse kinematic problem is 'ill-posed'.

Ill-posed problems do not have a physical sense. Rather, we have to define what the solution of an inverse kinematic problem means. Usually solutions of kinematic inversion are based on the criterion of the best fit of between calculated and observed reflection traveltimes. These solutions might not be unique and/or might not depend continuously on the data.

By itself, a good fit does not guarantee that an inverted model is correct. Kinematic inversion may lead to construction of several subsurface models with significantly different geological meaning, all of which fit the observed data equally well. Using a more complex model parameterization may lead to a better fit of calculated and observed traveltimes but it does not guarantee construction of a better subsurface model.

Although the problem of uniqueness and stability of the inverse kinematic problem is a hotly debated issue, most papers discuss this problem assuming that the model parameterization is adequate. Such an assumption is not realistic or verifiable in most case. Issues like the effect of the offset range, type of regularization, and concrete inversion algorithm are largely irrelevant for practical inverse problems because they invariably do not have solution (if by solution we mean an Earth model). The most difficult and the most important question of ill-posed problems is – how close is the estimated model to reality? It is our profound belief that unless we find a way to answer this question, all the existing methods of velocity model building are little more than a set of recipes from a cooking book.

Although a priori information may play an important role in stabilizing and constraining our solutions, in practice a priori information of the Earth's interior is poor. Most theoretical models are based on the Gaussian assumption of noise. Thus, such solutions are limited by the least-squares method which, however useful it may be in practical computations, is still based on an assumption. Limited knowledge of statistical properties of the Earth restricts the effectiveness of the statistical procedures. Numerically incorporating this limited knowledge into an objective function is still a big challenge.

We stress that the ill-posedness of kinematic inverse problems is fundamental and does not depend on a particular type of algorithm. It does not even depend on the approach underlying the algorithms (probabilistic or deterministic). In the numerical examples presented, we offer an (at least partial) approach to a well-posed solution. It relies on constructing a well-posed extension of the kinematic inverse problem by making additional assumptions and validating them.

We strongly believe that geophysicists like wrestlers have to honestly address issues like these, no matter how difficult, as in the Hamburg Score.

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