Optimal anti-aliasing for ray-based kirchhoff depth migration

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Summary

Operator/Imaging aliasing introduced in Kirchhoff migration is often tackled by trace tapering, aperture truncation or time and offset-variant filtering. The latter approach is the most suitable. However, most implementations and published results using this technique are derived for Kirchhoff time migration and assume a constant velocity media. In this paper, we introduce an anti-aliasing filter for ray-based pre-stack depth migration and for general heterogeneous velocity models. We illustrate the benefits of such a scheme on a numerical example.

Introduction

Three kinds of spatial aliasing can occur during the migration process, all leading to poor and ambiguous images. The three categories are: data, image and operator migration aliasing and have been extensively discussed by Lumley et al. (1994) and Biondo (2001). In this study only the operator migration aliasing is considered. Dealing with this aliasing effect is still an open problem and the most basic solutions are: data interpolation and aperture or migration dip truncation. However, the drawback of the latter technique is the suppression of steeply dipping events, while the former one can lead to an unpractical and expensive scheme. Gray (1992) incorporated explicitly the Nyquist criteria in the Kirchhoff migration. His scheme involves a time and offset-variant filter applied on the fly during migration and needs extra pre-computed filtered copies of the input trace. This technique is still the most adequate solution for Kirchhoff migration operator aliasing. Yet, all the proposed anti-aliasing filter solutions and associated results have been established for Kirchhoff time imaging (Gray, 1992; Lumley et al., 1994; Abma et al., 1999; Wang, 2000; Biondo, 2001; Zhang et al., 2001). Therefore, these solutions are based on a constant velocity hypothesis for a given trace time sample and imaging point.

In this paper, we propose to overcome the problem of migration operator aliasing in the depth domain. This extension works for a general heterogeneous velocity model and for a common offset ray-based prestack depth migration. We used the time-offset variant filter of Gray (1992), but here an optimal criteria for the high-frequency filtering is now established for the depth domain and for post and pre-stack migration algorithms.

Method

The discrete formula for Kirchhoff prestack depth migration can be summarized as :

$$I(\mathbf{x},h)=\sum_{M_i}\Delta M.D[M_i,h,T(M_i,h,\mathbf{x})],$$

where M_i, ΔM, h and $\mathbf{x}$ denote respectively the current CMP position, CMP sampling step, traces offset and depth imaging point. D is the time differentiated seismic reflection data, appropriately shifted by the two-way travel time $T(M_i,h,\mathbf{x})$ and corrected by an amplitude preserving weight to give us a quantitative recovery of the reflectivity function.

Roughly speaking, operator aliasing occurs when the migration summation path is too steep compared with the trace spacing and frequency content. On the other side, the continuous Kirchhoff migration summation

$$I(\mathbf{x},h)=\int dM.D[M,h,T(M,h,\mathbf{x})],$$

is alias free if the input seismic data is unaliased and the output image is sufficiently sampled. If the former condition is satisfied, data could be reconstructed between discrete positions by the use of Shannon theorem. However, unlike Zhang (2001) method, we perform data reconstruction along the dip of the local seismic events. Thus, we can write :

$$D[M,h,t]=\sum_{M_i}D[M_i,t+p_{data}(M-M_i)].\mathrm{sinc}[\frac{M-M_i}{\Delta M}].$$

At this stage, we have introduced the hypothesis that seismic reflection data are made by pieces of locally coherent events with dips p_{data}. Let us now introduce those interpolated data in the continuous Kirchhoff migration summation to obtain

$$I(\mathbf{x},h)=\sum_{M_i}\Delta M\int dt.D[M_i,t].F[t,M_i]$$

where :

- $F[t,M_i]=\int dM\frac{1}{\Delta M}\delta[t-\theta(\mathrm{M},\mathrm{M_i})].\mathrm{sinc}[\frac{M-M_i}{\Delta M}]$
- $\theta(\mathrm{M},\mathrm{M_i})=T(M_i,h,\mathbf{x})-p_{data}(M-M_i)$.

The kernel $F[t,M_i]$ can be simplified further using a variable change and first order Taylor expansion to get

$$F[t,M_i]=2\left(\frac{1}{2.\Delta M.}\frac{1}{\frac{\partial\theta}{\partial M}(M=M_i)}\right).$$

$$\times\text{sinc}\left[2\left(\frac{1}{2.\Delta M.}\frac{1}{\frac{\partial\theta}{\partial M}(M=M_i)}\right)\left(t-T(M_i,h,\mathbf{x})\right)\right]$$

We are now ready to recognize in the last expression a low-pass filter with the high frequency limit

$$f_{\max}(M_i,h,x)=\frac{1}{2.\Delta M\frac{\partial\theta}{\partial M}(M=M_i)}$$

$$=\frac{1}{2.\Delta M\left[p_{ray}(M_i,h,\mathbf{x})-p_{data}(t,M_i)\right]}$$

$p_{ray}(M_i,h,\mathbf{x})$ is the sum of horizontal component of slowness vector at source and receiver position. This quantity is computed numerically by paraxial ray tracing. On the other side, $p_{data}(t,M_i)$ has to be extracted from data by local slant stacking (Chauris, 2000). In this case, the proposed scheme will perform an optimal anti-aliasing protection without over-filtering or damaging amplitudes of steep dipping structures. Another solution is to set $p_{data}(t,M_i)$ with some a priori information. The simplest one is to choose it equal to zero and leads to a sub-optimal anti-aliasing criteria.

Application

A first test has been done on a simple canonical synthetic model with a moderate complexity. This model presents lateral and vertical velocity variations and dipping/curved reflectors. Figure 1. illustrate a zoom of one common offset synthetic data with offset equal to 100 m. As our concern here was to benchmark the anti-aliasing criteria, the true velocity model was used for depth migration.
Figure 2. presents depth imaging results around one reflector without any anti-aliasing protection. Coherent noise due to operator aliasing is more or less visible in this example. Figure 3. presents PSDM results using sub-optimal anti-aliasing criteria. This figure shows that the standard sub-optimal anti-aliasing criteria has significantly succeeded in removing all coherent noise. However, we can denote a severe amplitude and frequency contents losses. On the other side, PSDM result with the optimal anti-aliasing criteria shows better achievement of coherent noise removal and amplitude/frequency preservation.

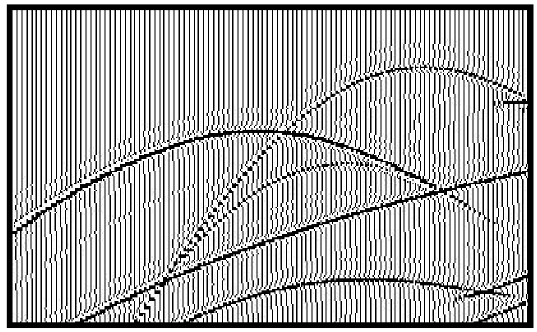

Figure 1 : Zoom on common offset (100 m) unmigrated prestack data.

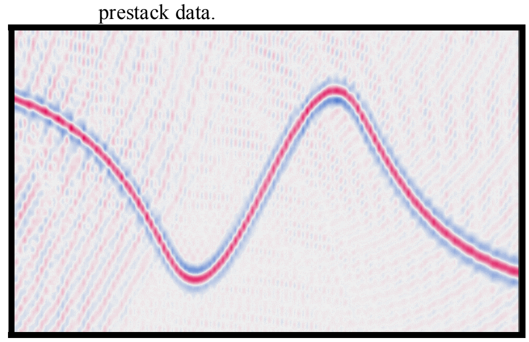

Figure 2 : PSDM result without anti-aliasing protection.

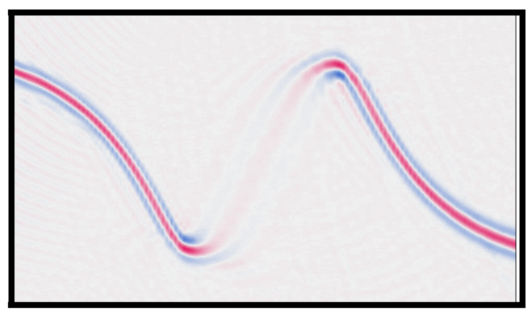

Figure 3 : PSDM result using classical sub-optimal anti-aliasing filtering.

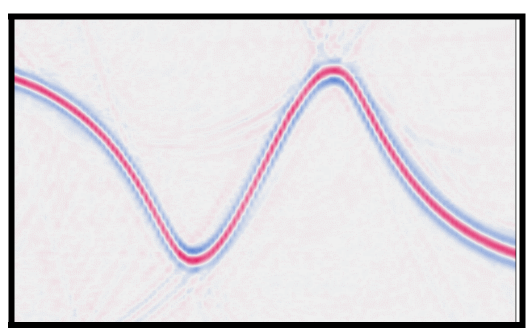

Figure 4 : PSDM result using optimal anti-aliasing filtering.

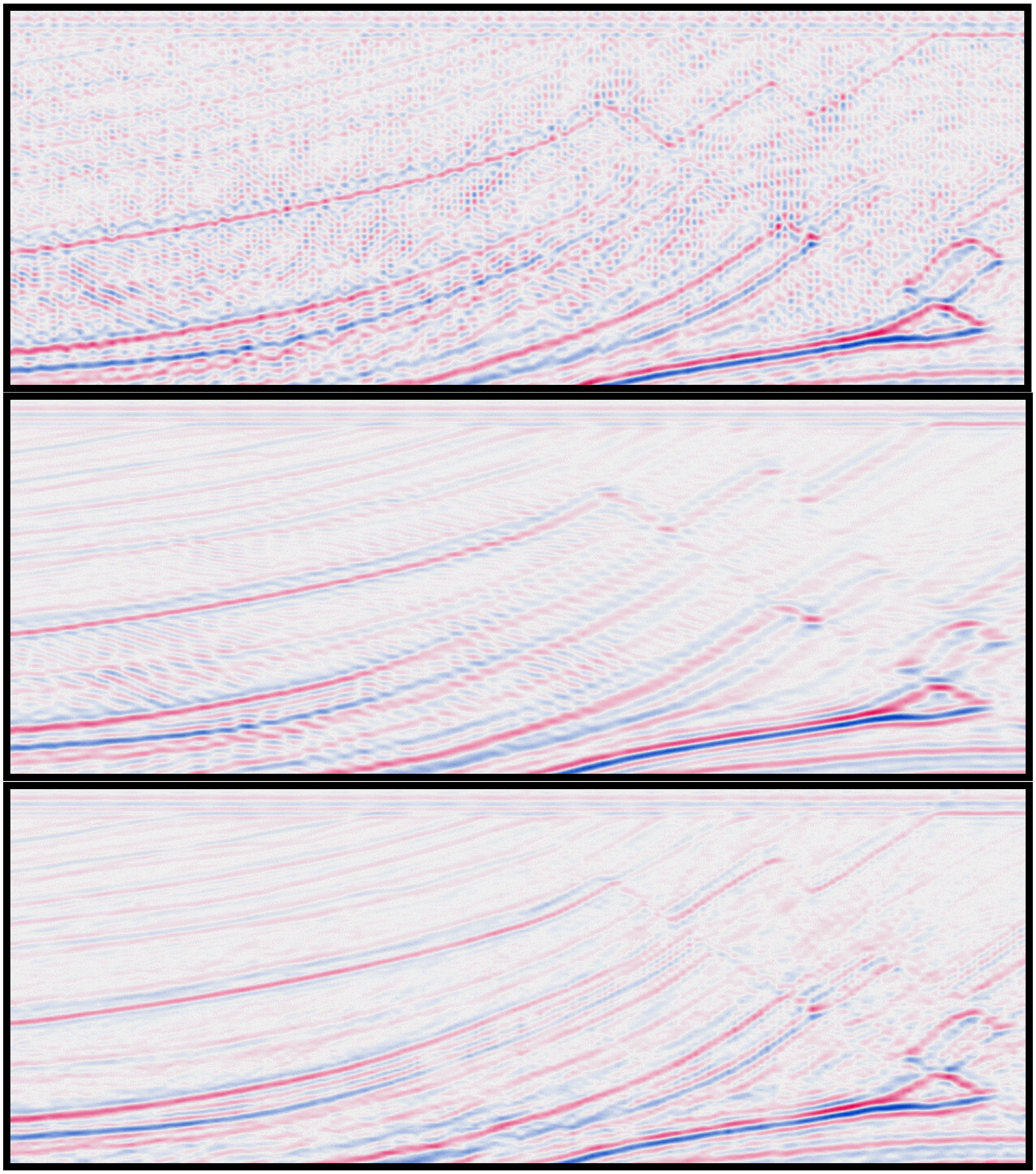

Figure 4 : Comparison of PSDM crude result (in the top) without any anti-aliasing correction, PSDM result (in the middle) using classical sub-optimal anti-aliasing criteria and finally in the bottom the PSDM result using optimal anti-aliasing criteria.

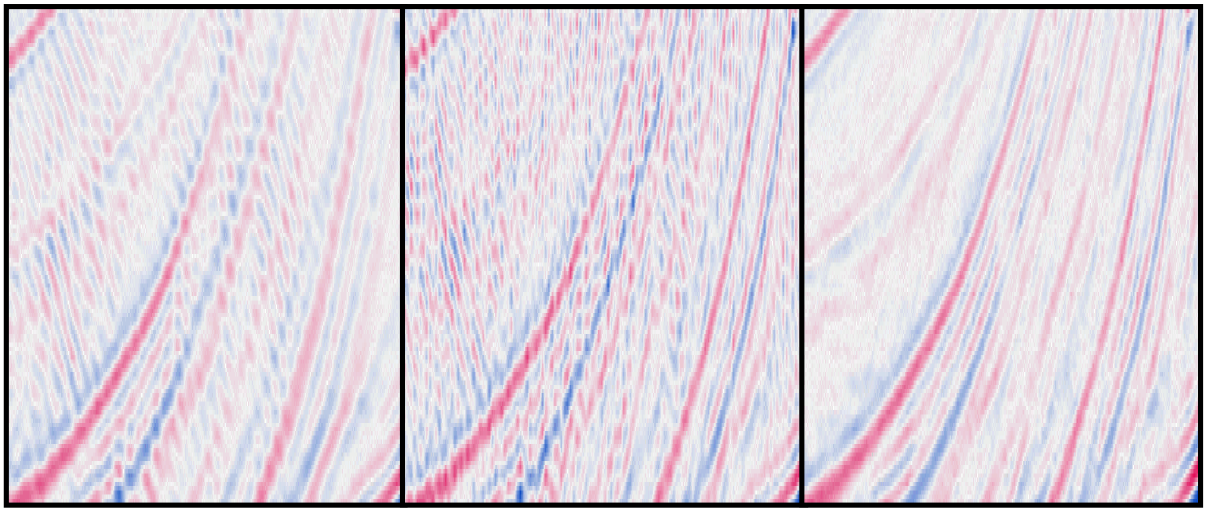

Figure 5 : Close-up of crude PDSM result without anti-aliasing filtering (in the middle) in comparison to PSDM result using standard anti-aliasing criteria (on the left) and to PSDM result using optimal anti-aliasing filtering criteria (on the left).

A second test was carried out using a more complicated synthetic dataset. The model has been built from the Marmousi model with some added horizontal layers in the top and the bottom of the model. 2.5D Ray+Kirchhoff modeling was used to generate synthetic prestack data using 25 m sampling step for shot and sources positions. Figure 4 illustrate the comparison of crude (without anti-aliasing filtering) depth imaging result results versus the sub-optimal anti-aliasing filtering and versus the optimal criteria. Sub-optimal anti-alising filtering improve imaging results and significantly reduce operator aliasing noise. However, the optimal anti-aliasing criteria leads to a better preservation of amplitude and frequency contents even for steep dipping reflectors.

Conclusions

The improvement when taking into account the operator migration aliasing has been already demonstrated for Kirchhoff time migration. Following the study of Zhang (2001), we have established an improved criteria for the anti-aliasing filtering in prestack depth migration. With this work, we have gained insight in the over-filtering and amplitude reduction when the classical anti-aliasing filter is applied. The method uses paraxial ray tracing and the time-offset variant traces filtering of Gray (1992). Therefore, it is valid for arbitrary depth migration velocity models and reflector shapes. We have presented some results of a Kirchhoff common-offset migration in the depth domain, with a highly attenuated operator aliasing noise. Further work should be done for reliable extraction of the slope of locally coherent events in the 3D real prestack data.

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